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A self-similar unsteady flow with conjugated heat transfer

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Abstract

In this paper an exact analytical self-similar solution of the thermo-fluid dynamic field arising in an impulsively accelerated flow over a flat plate is proposed. The plate is considered of infinite thickness and the thermal field is computed both in the fluid and in the solid with the temperature and the heat flux unknown at the solid–fluid interface (conjugated heat transfer). The values of the initial temperatures in the solid and in the fluid are different constants. The solution, obtained in the incompressible case, is extended to compressible flows by the Stewartson–Dorodnitsin transformation. The influence of the non-dimensional parameters governing the phenomenon is discussed with particular emphasis to the simple expressions of the interface temperature and heat flux.

Keywords: Conjugated heat transfer; Unsteady flow; Self-similar solution

1. Introduction

In thermo-fluid dynamic problems, the boundary conditions for the thermal field are usually assigned at the solid walls in terms of temperature or heat flux. However, both temperature and heat flux at the solid wall are in general unknowns and should be determined by simultaneous and coupled solutions of the thermo-fluid dynamic equations in the fluid and the energy equation in the solid enabling the continuity of the temperature and heat flux at the solid–fluid interface. This problem is known in literature as Conjugated heat transfer after Perelman [1]. These phenomena are relevant in many applications such as aerospace and cooling technologies, see [2] for a review on the subject and further literature. Moreover conjugated effects should be carefully considered for avoiding misleading conclusions in experimental or numerical analysis involving thermal effects, see [3] for an example.

Because of the complexity of the problem, conjugated heat transfer is usually analyzed by numerical methods [4] or by approximate solutions [5,6]. Fundamental theoretical works were proposed in [7,8], but they did not provide quantitative results for practical applications. However, we recently showed the possibility to obtain, at least for a simple geometry, an analytical exact solution [9]. In that paper the solution was proposed in the case of an impulsively accelerated flow from rest to a constant speed over an infinite plate of finite thickness in the case of imposed temperature or adiabatic condition on the unwetted side of the plate, also including the effects of dissipation of kinetic energy in the fluid. The solutions were obtained by applying the Laplace transform technique and could be extended to the case of compressible flow by the Stewartson–Dorodnitsin transformation.

A very interesting and more general result of the paper is that the exact coupling condition of the thermal field in the fluid and in the solid can be obtained with simple algebraic equations instead of the complex integral equations proposed in previous theoretical works. This result gives the possibility to search further analytical solutions of conjugated problems of practical interest by adopting standard

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Nomenclature

E	$U_\infty^2/(c_p T_\infty)$
Ec	Eckert number
K	$\sqrt{2/Pr - 1}$
L	reference length
Nu	Nusselt number
Nu_0	Nusselt number of a plate with infinitely small thickness
Pr	Prandtl number
Re	Reynolds number
T	temperature
U, V	Stewartson–Dorodnitsin transformation of velocity components
U_∞	freestream velocity imposed at initial time
$Z_h, Z_p, \overline{Z}$	self similar functions
c_p	specific heat of the fluid at constant pressure
c_s	specific heat of the solid
h_f, h_s	scale factors of self-similar solutions in the fluid and in the solid
p	$(\lambda_f/\lambda_s)\sqrt{Re}$
t	time
t_{fs}	$\alpha_s/(U_\infty L)$
u, v	velocity components
y	spatial coordinate in the fluid normal to the solid–wall interface
Λ	$\sqrt{\rho_f c_p \lambda_f}/\sqrt{\rho_s c_s \lambda_s}$, thermal activity ratio
$\Omega_h, \Omega_p, \bar{\Omega}$	time scale factors in self-similar solutions
α	thermal diffusivity
η	non-dimensional spatial coordinate in the fluid normal to the solid–wall interface
λ	thermal conductivity
ν	kinematic viscosity of the fluid
ρ	density
θ	non-dimensional temperature in the fluid
$\bar{\theta}$	non-dimensional temperature in the solid
θ_{aw}	adiabatic wall temperature over a plate of zero-thickness in impulsive Rayleigh flow
τ	non-dimensional time
ξ	spatial coordinate in the solid normal to the solid–wall interface
$\bar{\xi}$	non-dimensional spatial coordinate in the solid normal to the solid–wall interface
ζ	similarity variable

Subscripts

e	position in the solid infinitely far from the solid–fluid interface
f	fluid
s	solid
w	solid–fluid interface
∞	freestream condition

and straightforward techniques for the solution of partial differential equations.

In this note we verify the possibility to obtain unsteady self-similar solutions when the plate is of infinite thickness, as suggested by the analysis in [9]. This assumption can be applied for very small time values or when the length scale is really large such as, for instance, the thermal boundary layer of the atmosphere coupled with the ground in environmental applications. Instead of applying the Laplace transform technique we here derive the solution by the standard search for self-similar solutions. We propose the solution in the case of assigned temperature in the solid infinitely far from the solid–fluid interface. The analytical solution is straightforward and in explicit form. The results are discussed by analyzing the impact of the conjugated effects (also comparing them with the solution for the case of plate of infinitely small thickness) and by showing the obtained temperature profiles, both in the fluid and in the solid, in terms of the involved parameters.

2. The physical problem and solution

We consider an infinite plate (in both directions) of infinite thickness, wetted by a fluid on one side which is impulsively accelerated from rest to a constant speed U_∞ at the initial time $t = 0$. The problem is one-dimensional in space; the thermo-fluid dynamic field in the fluid depends on t and on y, the spatial coordinate orthogonal to the plate, with the origin placed at the solid–fluid interface and pointing towards the fluid. The temperature field in the solid depends on t and on ξ, the spatial coordinate orthogonal to the plate, with the origin placed at the solid–fluid interface and opposite direction with respect to y, see Fig. 1.

The initial temperature field in the fluid $T_f(y, t)$ is uniform, with $T_f(y, 0^-) = T_\infty$. In the solid the initial temperature field $T_s(\xi, t)$ is also uniform with a different temperature value $T_s(\xi, 0^-) = T_e$. We assume that an incompressible, viscous and laminar flow with constant

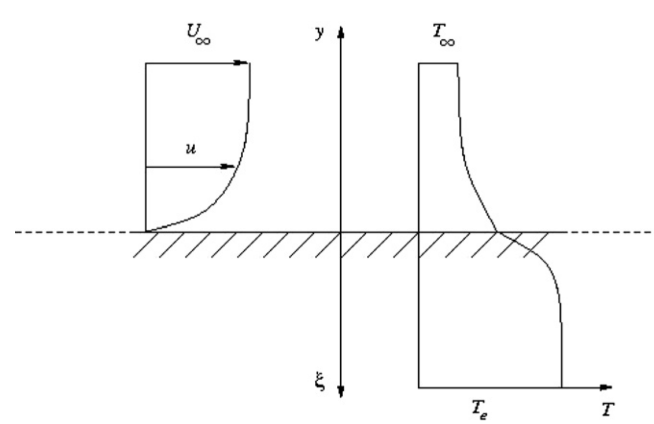

Fig. 1. Sketch of the physical problem for $t > 0$.

properties (kinematic viscosity ν and thermal conductivity λ_f) arises, therefore the dynamic field is not coupled with the thermal one. The boundary conditions for the velocity are the matching with the freestream value for $y \to \infty$ and the no-slip condition on the plate. In this case the solution of the Navier–Stokes equations is given by the well known Rayleigh flow, see [10], p. 137:

$$u = \operatorname{erf} \zeta, \tag{1}$$

where u is the non-dimensional velocity component parallel to the plate referenced to U_∞, $\operatorname{erf} z = (2/\sqrt{\pi}) \int_0^z e^{-\chi^2} d\chi$ specifies the error function [11], and $\zeta = y/\sqrt{4\nu t} = \eta/(\sqrt{4\tau})$ is the similarity variable of the dynamic field with $\eta = y\sqrt{U_\infty/(\nu L)} = y\sqrt{Re}/L$, $\tau = tU_\infty/L$ (L is a reference length and Re the Reynolds number).

Denoting $\theta = (T_f - T_\infty)/T_\infty$ and $\bar{\theta} = (T_s - T_\infty)/T_\infty$ respectively the non dimensional temperature in the fluid and in the solid, the energy equations in the fluid and in the solid are:

$$\frac{\partial \theta}{\partial \tau} - \frac{1}{Pr}\frac{\partial^2 \theta}{\partial \eta^2} = E\left(\frac{\partial u}{\partial \eta}\right)^2, \tag{2a}$$

$$\frac{\partial \bar{\theta}}{\partial \tau} = t_{fs}\frac{\partial^2 \bar{\theta}}{\partial \bar{\xi}^2}, \tag{2b}$$

where $E = U_\infty^2/(c_p T_\infty)$ (c_p is the specific heat at constant pressure of the fluid), Pr is the Prandtl number, $\bar{\xi} = \xi/L$ and $t_{fs} = \alpha_s/(U_\infty L)$ (α_s is the thermal diffusivity in the solid). The parameter E is strictly connected with the Eckert number (Ec) since $Ec = ET_\infty/\Delta T_{ref}$. The initial conditions are $\theta(\eta, 0) = 0$ for the fluid and $\bar{\theta}(\bar{\xi}, 0) = \bar{\theta}_e$ for the solid. A boundary condition for the fluid is $\theta(\eta, \tau) \to 0$ for $\eta \to +\infty$. Similarly, for the solid we have $\bar{\theta}(\bar{\xi}, \tau) \to \bar{\theta}_e = T_e/T_\infty - 1$ for $\bar{\xi} \to +\infty$. Finally the continuity of the temperature and of the heat flux at the solid–fluid interface must be imposed:

$$\theta(0, \tau) = \bar{\theta}(0, \tau), \tag{3a}$$

$$p\frac{\partial \theta(0, \tau)}{\partial \eta} = -\frac{\bar{\partial}\theta(0, \tau)}{\partial \bar{\xi}}, \tag{3b}$$

where $p = (\lambda_f/\lambda_s)\sqrt{Re}$ (λ_s is the thermal conductivity in the solid).

The lack of a length scale suggests to look for self-similar solutions of these equations. It is convenient to study the linear energy equation in the fluid in the form $\theta = \Omega_h(\tau)Z_h(\zeta_f) + \Omega_p(\tau)Z_p(\zeta_f)$ with $\zeta_f = \eta/h_f(\tau)$, where the first and the second term are respectively the solution of the associated homogeneous equation and a particular solution of the complete equation. Similarly the solution of the homogeneous energy equation in the solid is put in the form $\bar{\theta} = \bar{\Omega}(\tau)\bar{Z}(\zeta_s)$ with $\zeta_s = \bar{\xi}/h_s(\tau)$. Then the analysis of the self-similar solutions follows by the standard techniques. In this case the boundary conditions for the fluid and the solid can be satisfied together with the matching conditions Eq. (3) only if $\Omega_h(\tau)$, $\Omega_p(\tau)$ and $\bar{\Omega}(\tau)$ are constant values, which implies that the interface temperature does not depend on time. Therefore Eq. (3a) reduces to $\theta(0, \tau) = \bar{\theta}(0, \tau) = \theta_w = T_w/T_\infty - 1$, which is the temperature at the solid–fluid interface.

In the case of assigned constant wall temperature both the solutions in the fluid and in the solid are well known. In the fluid the temperature field is given by (see [13] for a detailed analysis of this solution):

$$\theta(\zeta_f) = \theta_w + (\theta_{aw} - \theta_w)\operatorname{erf}\zeta_f - \frac{2}{K}\frac{E}{\sqrt{\pi}}\int_0^{\zeta_f} e^{-\chi^2}\operatorname{erf}(K\chi)\,d\chi, \tag{4}$$

where $\zeta_f = \sqrt{Pr}\,\zeta$, $K = \sqrt{2/Pr - 1}$ and

$$\theta_{aw} = 2E\frac{\arctan K}{\pi K} \tag{5}$$

is the adiabatic wall temperature in the case of impulsive Rayleigh flow over a plate of zero-thickness.

The temperature field in the solid is

$$\bar{\theta}(\zeta_s) = (\theta_w - \bar{\theta}_e)(1 - \operatorname{erf}\zeta_s) + \bar{\theta}_e, \tag{6}$$

where $\zeta_s = \xi/\sqrt{4\alpha_s t} = \bar{\xi}/(\sqrt{4t_{fs}\tau})$.

Both solutions are in self-similar form with similarity variables ζ_f and ζ_s. θ_w, the temperature at the solid–fluid interface, is the only unknown; it can be computed imposing that Eq. (3b) is satisfied. Taking into account for Eqs. (4) and (6), Eq. (3b) reduces to

$$\Lambda(\theta_{aw} - \theta_w) = \theta_w - \bar{\theta}_e, \tag{7}$$

where the *thermal activity ratio* $\Lambda = \sqrt{\lambda_f\rho_f c_p}/\sqrt{\lambda_s\rho_s c_s} = p\sqrt{t_{fs}}\sqrt{Pr}$ has been introduced. Λ is the ratio between the thermal effusivities in the fluid and in the solid (ρ_f and ρ_s are the density in the fluid and in the solid, c_s is the specific heat in the solid). Then, the interface temperature is:

$$\theta_w = \frac{\Lambda}{1+\Lambda}\theta_{aw} + \frac{1}{1+\Lambda}\bar{\theta}_e. \tag{8}$$

3. Analysis of the results

3.1. Interface temperature

Eq. (8) shows that the *interface temperature is constant even if the problem is unsteady* and is given by an average of the adiabatic wall temperature in the fluid and the asymptotic temperature in the solid weighted by the thermal activity ratio Λ.

This result is surprising at a first sight. However the present analysis is in agreement with the results proposed in [9] for the plate of finite thickness, where the interface temperature is not constant with time. Nonethless the time derivative of the interface temperature is zero for $\tau \to 0^+$ and its value is coincident with Eq. (8): the interface temperature becomes constant when the thickness of the plate becomes infinite.

In addition, there is an analogy with a different problem: the case of two semi-infinite solids with a flat interface and different temperatures at the initial time, see for instance [12], pp. 87–88; the difference is that here we have a production term in the energy equation of the fluid due to the dissipation of kinetic energy. In both cases the solutions are self-similar and the interface temperature is constant with time even if the problem is unsteady. The presence of a transition region for the interface temperature requires a finite thickness of the solid.

Eq. (8) provides the wall temperature for $t > 0$, which, on the other side, is 0 for $t < 0$, therefore there is a jump in time at $t = 0$; this discontinuity is driven by the presence, at the initial time, of a singularity in the temperature distribution and in the dynamic field. θ_w is ruled by the parameter Λ. Some typical values of Λ for different solid–fluid combinations are shown in Table 1. In the table Λ ranges from $\approx 10^{-4}$ to 10.

The temperature jump in the fluid at the initial time is given by Eq. (8). Defining $\Delta\theta_w = \theta_w - \bar{\theta}_e$, the temperature jump in the solid is

$$\frac{\Delta\theta_w}{\bar{\theta}_e} = \frac{\Lambda}{1+\Lambda}\left(\frac{\theta_{aw}}{\bar{\theta}_e} - 1\right). \tag{9}$$

Eqs. (8) and (9) show that the initial temperature jump only depends on Λ, on θ_{aw}, therefore on Ec and Pr, and on T_e/T_∞. In particular, when $Ec = 0$ (the dissipation of kinetic energy is negligible) $\theta_{aw} = 0$ and the interface temperature does not depend on the dynamic field in the fluid, but only on its physical properties (through Λ).

The conjugated effects on the interface temperature can be easily quantified by Eq. (9). This relation is plotted in Fig. 2 with results evidenced for typical solid–fluid combinations. The conjugated effects become relevant for large values of Λ.

If we consider the case in which, at the initial time, the fluid is not accelerated from rest: $u(\eta, \tau) = 0\ \forall\tau > 0$, the energy field in the fluid is also governed by the heat equation and the temperature field has the same form of the solution in the solid, Eq. (6). In this case the continuity of the heat flux at the solid–fluid interface provides

$$\theta_w = \frac{1}{1+\Lambda}\bar{\theta}_e. \tag{10}$$

This relation shows that the interface temperature arising when the fluid is impulsively accelerated from rest is exactly equivalent to the temperature arising at the interface between two semi-infinite solids with flat interface and at different initial temperatures, provided that the initial temperature in the fluid is replaced by the adiabatic wall temperature. An interesting consequence is that, during this self-similar stage, conduction is more efficient than convection in a cooling system.

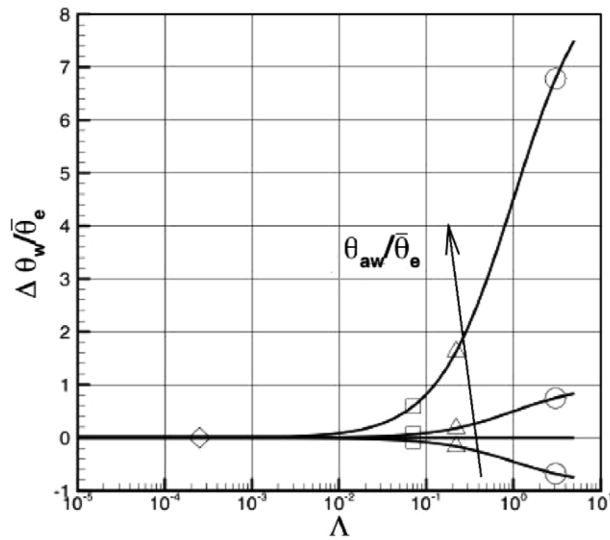

Fig. 2. $\Delta\theta_w/\bar{\theta}_e$ versus Λ for $\theta_{aw}/\bar{\theta}_e = 0.1, 1, 2, 10$. ◇: air–aluminium; □: water–aluminium; △: water–iron; and ○: mercury–glass.

3.2. Interface heat flux

The heat flux study is proposed in terms of the Nusselt number $Nu = -\frac{L}{\Delta T_{ref}}\left(\frac{\partial T}{\partial y}\right)_w$ ($\Delta T_{ref} = T_e - T_\infty$ and $L = U_\infty t$) given by:

$$\frac{Nu}{Re} = -\frac{\sqrt{Pr}}{2\bar{\theta}_e}\left(\frac{\partial\theta}{\partial\zeta_f}\right)_w = \frac{\sqrt{Pr}}{\sqrt{\pi}}\frac{(\theta_w - \theta_{aw})}{\bar{\theta}_e} = \frac{\sqrt{Pr}}{\sqrt{\pi}(1+\Lambda)}\left(1 - \frac{\theta_{aw}}{\bar{\theta}_e}\right). \tag{11}$$

For $\bar{\theta}_e > 0$ the fluid is cooling or heating the plate respectively for $\theta_{aw} < \bar{\theta}_e$ and $\theta_{aw} > \bar{\theta}_e$. For $\bar{\theta}_e < 0$, as obvious, the fluid is always heating the plate. Eq. (11), compared with the analogous relation proposed in [13] for the plate of infinitely small thickness ($\bar{\theta}_e$ in place of θ_w), allows for an exact evaluation of conjugated effects on the heat flux. In particular, the conjugated effects reduce the heat flux of a factor $(1 + \Lambda)$. This result is evidenced in Fig. 3. Nu/Nu_0 is plotted versus Λ, where Nu_0 specifies the Nusselt number in the case of infinitely small thickness of the plate. In terms of both temperature and heat flux at the wall, the

Table 1
Values of the thermal activity ratio Λ for different solid–fluid combinations

	Mercury ($\times 10^{-1}$)	Air ($\times 10^{-4}$)	Water ($\times 10^{-2}$)	Glycerin ($\times 10^{-2}$)	Light oil ($\times 10^{-1}$)
Aluminium	1.79	2.57	7.09	4.20	0.210
Iron	5.51	7.92	21.9	12.9	0.647
Glass	27.9	40.2	111	65.7	3.28
Oak wood	61.1	87.9	243	144	7.18
Concrete	29.1	41.9	116	68.5	3.42

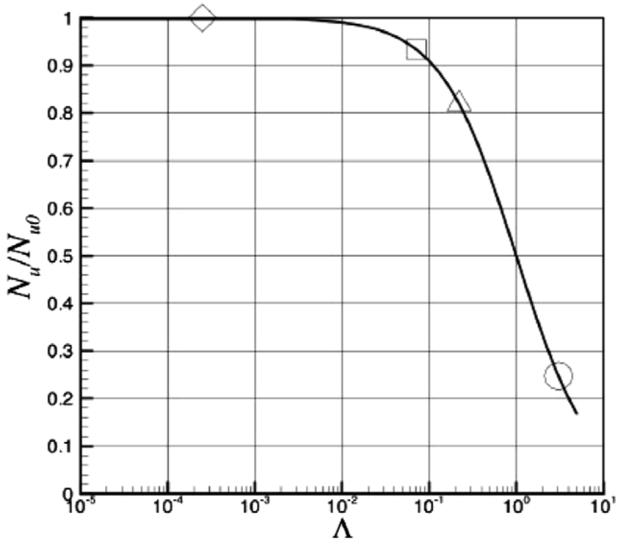

Fig. 3. Nu/Nu_0 versus Λ. ◇: air–aluminium; □: water–aluminium; △: water–iron; and ◯: mercury–glass.

conjugated effects are negligible for $\Lambda \ll 1$, but, again, they become more and more significant as Λ increases.

3.3. Temperature profiles

The self-similar temperature profiles in the fluid and in the solid, providing the temperature for whatever value of time and space position, are plotted in Fig. 4 for different values of Λ. When the conjugated effects are negligible ($\Lambda \to 0$), the temperature in the solid is approaching an uniform distribution $\bar{\theta} = \bar{\theta}_e$. For $\zeta_s = 1.82$ it is $\text{erf}\,\zeta_s = 0.99$, therefore the thickness in the solid in which temperature variations are significant for a given Λ ($> 1\%$) is $b = 3.64\sqrt{\alpha_s t}$. This expression also gives the possibility to identify, in the case of a plate of finite thickness, the time interval in which the approximation of infinite thickness is valid.

The influence of the Eckert number $Ec = E/\bar{\theta}_e$ is shown in Fig. 5. As Ec increases, the typical effects of the dissipation of kinetic energy appear: the plate is heated even for $T_\infty < T_e$.

3.4. Compressible flow

Also in this case compressibility effects can be taken into account by the Stewartson–Dorodnitsin transformation, see [14], p. 246. Assuming $v/v_\infty = \lambda_f/\lambda_\infty = \rho_\infty/\rho_f = T_f/T_\infty$ the *unsteady* transformation is

$$\eta = \frac{\sqrt{Re}}{L}\int_0^y \frac{\rho_f}{\rho_\infty}\,d\bar{y}, \quad U = u, \quad V = \frac{\rho_f}{\rho_\infty}v + u\frac{\partial\eta}{\partial\tau} + u\frac{\partial\eta}{\partial X}, \tag{12}$$

where v is the non-dimensional velocity component normal to the wall and X is the non-dimensional spatial coordinate along the plate. U and V are the transformed velocity com-

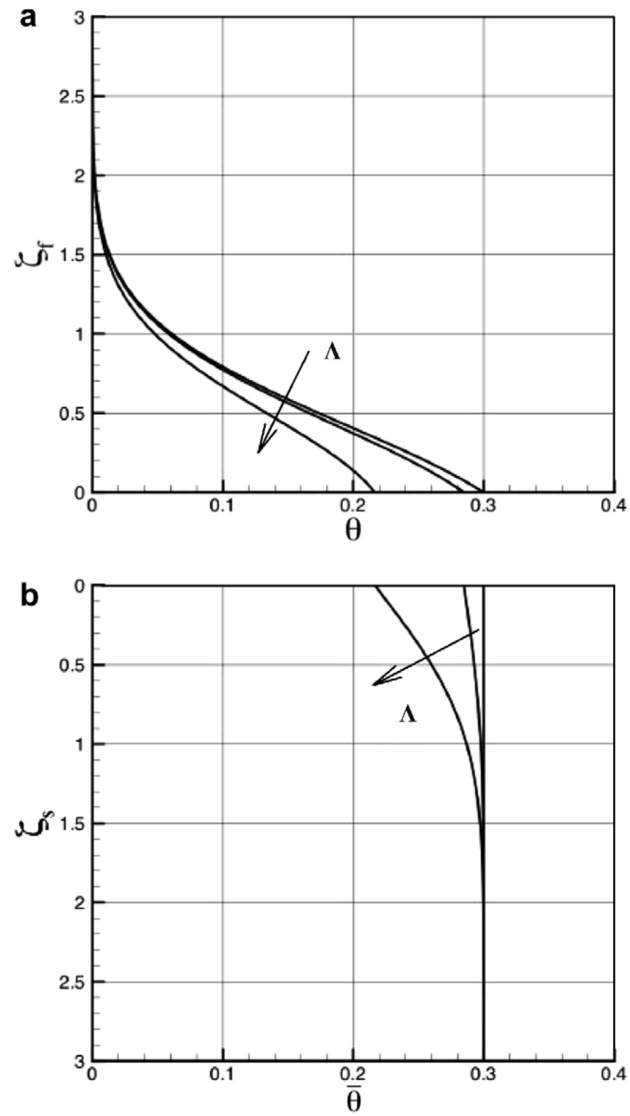

Fig. 4. Temperature distribution in the fluid θ (a) and in the solid $\bar{\theta}$ (b) for $\Lambda = 0.01, 0.1, 1$. $\bar{\theta}_e = 0.3$, $E = 0.3$, $Pr = 0.72$. $\zeta_f = \sqrt{Pr}\,\eta/\sqrt{4\tau}$, $\zeta_s = \bar{\xi}/\sqrt{4t_{fs}\tau}$.

ponents. This transformation decouples the dynamic field from the thermal one. U is again given by the error function, although the physical velocity distribution along y is modified by the relations (12). The V component in the transformed plane is still zero, while the physical velocity component v is given by

$$v = -u\frac{\partial\eta}{\partial\tau}\frac{\rho_\infty}{\rho_f}. \tag{13}$$

The energy equation has the same expression of Eq. (2a) with U in place of u and $E = (\gamma - 1)M_\infty^2$, where γ is the ratio between the specific heats and M_∞ is the freestream Mach number. However, it should be noted that in this case the physical normal velocity is not zero and the flow is no more parallel, see [15,16] among others, for a detailed discussion of the compressible Rayleigh flow.

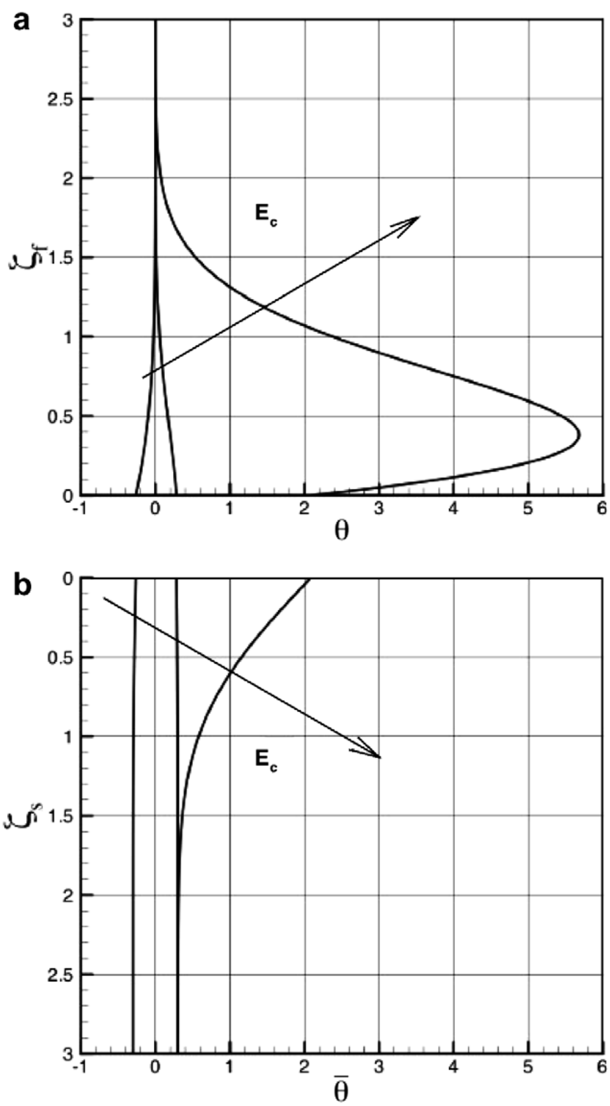

Fig. 5. Temperature distribution in the fluid θ (a) and in the solid $\bar{\theta}$ (b) for $Ec = -1, 1, 5$. $\Lambda = 0.1$, $Pr = 0.72$. $\zeta_f = \sqrt{Pr}\eta/\sqrt{4\tau}$, $\zeta_s = \bar{\xi}/\sqrt{4t_{fs}\tau}$.

4. Conclusions

Analytical solutions can provide a deeper understanding of physical phenomena; the effects of the parameters ruling the problem are explicit and they are useful test cases for the validation and verification of numerical methods. In addition, they can be applied to obtain a quick quantitative analysis looking at the local or asymptotic behavior of more complex fields.

Recent results showed the possibility to obtain simple algebraic conditions in order to exactly model the conjugated effects in thermo-fluid dynamic fields: hence, standard techniques for the solution of partial differential equations can be adopted for deriving analytical solutions of problems of practical interest. In the present case, an exact self-similar solution of the thermal field has been presented for a flow impulsively accelerated over an infinite plate of infinite thickness. It has a very simple explicit form. Present results have been obtained by the standard technique adopted for the search of self-similar solutions.

The temperature and the heat flux at the solid–fluid interface are constant with time; the presence of a transition at the solid–fluid interface requires a finite thickness of the solid.

The role of the thermal activity ratio Λ (depending on the thermal properties of the fluid and of the solid) and of the adiabatic wall temperature for the impulsive Rayleigh flow have been highlighted. The results show how conjugated effects play an important role when Λ is not small and that, at least in the present flow, there is an analogy with the problem of pure conduction when the asymptotic temperature in the fluid is replaced by the adiabatic wall temperature.

The proposed very simple expressions for the interface temperature and heat flux can be used in more general cases when the approximation of infinite thickness can be applied.

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